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[B.Sc - II year]

Paper - I, Unit - IV

(Matrix)

UNIT – 4

Matrices

Matrix is a rectangular arrangement in which we arrange some numbers in horizental and vertical lines are called rows and the vertical lines are called column of the matrix.

For Example →

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 1 \end{bmatrix} \end{array} \begin{array}{l} \rightarrow \text{row} \\ \\ \\ \end{array}$$

↓ column

Order Of A Matrix →

Order of a matrix is a way of representation of number of row and number of column present in matrix.

Example →

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}_{2\times3}$$

$$\boxed{\text{Order} = 2\times3}$$

Algebra Of matrices →

(i) Sum of two matrices → The sum of two matrices is possible only when the order of two matrices are same.

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Example → Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{2\times2}$ & $B = \begin{bmatrix} 3 & 5 \\ 6 & 1 \end{bmatrix}_{2\times2}$

Then, $A+B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{2\times2} + \begin{bmatrix} 3 & 5 \\ 6 & 1 \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 5 & 8 \\ 10 & 6 \end{bmatrix}_{2\times2}$

Note :- The substraction of two matrices is possible as same as for addition.

Multiplication of two matrices →

The multiplication of two matrices is possible when number of column in first matrix and the number of row in second matrix are same.

Example → $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}$ & $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}_{2\times3}$

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}_{2\times3}$

$= \begin{bmatrix} 1+6 & 2+2 & 3+4 \\ 3+12 & 6+4 & 9+8 \end{bmatrix} = \begin{bmatrix} 7 & 4 & 7 \\ 15 & 10 & 17 \end{bmatrix}$

Que. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 3 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ then what the multiplication of A&B.

Solⁿ → we have given –

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 3 & 2 \end{bmatrix}_{3\times2} \quad \& \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}_{2\times3}$$

then,

$$AB = \begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 7 & 12 \\ 23 & 16 & 27 \\ 9 & 12 & 13 \end{bmatrix}_{3\times3}$$

Transpose Of a Matrix →

let A be a matrix of any order if we change row column and column in row then get a new matrix called transpose of A. It is denoted by A^T or A

Example → $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}_{2\times2}$

$$\Rightarrow A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}_{2\times2}$$

Theorem → The transpose of a transposed matrix is matrix itself.

Proof → let A be a matrix of order $m\times n$.

i.e. $A = [a_{ij}]_{m\times n}$ —— (i)

we have to show $\boxed{(A^T)^T = A}$

Since $A = [a_{ij}]_{m\times n}$

On taking transpose both sides we get –

$$A^T = \{[a_{ij}]_{m\times n}\}^T$$

Thus the transpose of sum of the two matrices is the sum of their individual transpose.

Example → $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 5 & 8 \\ 2 & 5 & 6 \end{bmatrix}$

$$A+B = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 8 \\ 2 & 5 & 6 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 8 & 13 \\ 3 & 11 & 8 \end{bmatrix}$$

$$\Rightarrow (A+B)^T = \begin{bmatrix} 3 & 3 \\ 8 & 11 \\ 13 & 8 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 5 & 2 \end{bmatrix} \quad \& \quad B^T = \begin{bmatrix} 1 & 2 \\ 5 & 5 \\ 8 & 6 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 3 \\ 8 & 11 \\ 13 & 8 \end{bmatrix}$$

$$\boxed{(A+B)^T = A^T + B^T = \begin{bmatrix} 3 & 3 \\ 8 & 11 \\ 13 & 8 \end{bmatrix}}$$

Theorem → The transpose of product of two matrices is equal to product of their individual transpose but in reverse order.

Proof → Let $A = [a_{ij}]_{m \times n}$ —— (i)

and $B = [b_{jk}]_{n \times k}$ —— (ii) be two matrices.

We have to show –

$$\boxed{(A \cdot B)^T = B^T \cdot A^T}$$

On multiplying (i) & (ii) we get –

$$AB = [a_{ij}]_{m \times n} \; [b_{jk}]_{n \times k}$$

$$AB = [a_{ij} \cdot b_{jk}]_{m \times k}$$

$$AB = [C_{ik}]_{m \times k} \quad \forall \; a_{ij} \cdot b_{jk} = C_{jk}$$

$$(AB)^T = [C_{jk}]^T_{m \times k}$$

$$(AB)^T = [C_{ki}]_{k \times m} \quad \text{—— } (*)$$

Now from eqn (i) & (ii) we get –

$$A^T = [a_{ij}]^T_{m \times n} \quad \& \quad B^T = [b_{jk}]^T_{n \times k}$$

$$A^T = [a_{ji}]_{n \times m} \quad \& \quad B^T = [b_{kj}]_{k \times n}$$

So, $B^T \cdot A^T = [b_{kj}]_{k \times n} \cdot [a_{ji}]_{n \times m}$

$$= [b_{kj} \cdot a_{ji}]_{k \times m}$$

$$B^T A^T = [C_{kj}]_{k \times m} \quad \text{—— } (**)$$

from $(*)$ & $(**)$ –

$$\boxed{(AB)^T = B^T \cdot A^T}$$

Thus the transpose of product of two matrices is equal to product of their indivisual transpose but in reverse.